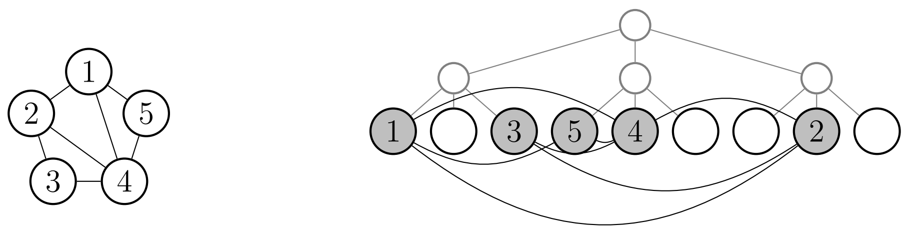
Data arrangement problem on binary regular trees

Let $G = (V, E)$, where $|V(G)| = n$ be a so-called *guest* graph and let $d \geq 2$ be fixed. Furthermore, let the so-called *host* graph be a d-regular tree T of height $h = \lceil \log_d n \rceil$ and let B be the set of its leaves. The *data arrangement problem on regular trees* (*DAPT*) asks for an arrangement ϕ that minimizes the objective value $OV(G, d, \phi)$.

$$OV(G, d, \phi) := \sum_{(u,v) \in E} d_T\big(\phi(u), \phi(v)\big), \tag{1}$$

where $\phi\colon V(G) \to B$ is an injective embedding and $d_T\big(\phi(u), \phi(v)\big)$ denotes the length of the unique $\phi(u)$-$\phi(v)$-path in the d-regular tree T.

Example 1 *Let $G = (V, E)$ be the guest graph depicted in Figure 1a and let $d = 3$. The height of the host graph T is $h = \lceil \log_d n \rceil = \lceil \log_3 5 \rceil = 2$.*

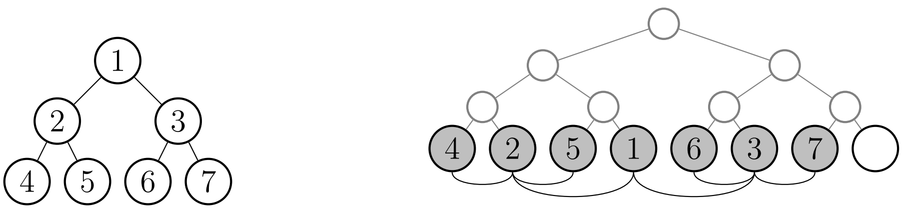


(a) Guest graph. (b) Host graph.

Figure 1: Example 1.

Let ϕ bet the arrangement depicted in Figure 1b. We have to count 2 for the edge $(4, 5)$ and 4 for all remaining edges. Thus $OV(G, 3, \phi) = 26$.

Example 2 *Let $G = (V, E)$ be the binary regular tree depicted in Figure 2a and let $d = 2$. The height of the host graph T is $h = \lceil \log_d n \rceil = \lceil \log_2 7 \rceil = 3$.*



(a) Guest graph. (b) Host graph.

Figure 2: Example 2.

Let ϕ bet the arrangement depicted in Figure 2b. We have to count 2 *for the edges* $(2,4)$ *and* $(3,6)$*,* 4 *for the edges* $(1,2)$*,* $(2,5)$ *and* $(3,7)$*, and* 6 *for for the edge* $(1,3)$*. Thus* $OV(G,2,\phi) = 22$*.*

Luczak and Noble [1] have shown that this problem is $\mathcal{NP}$-hard for every fixed $d \geq 2$. Çela, Schauer and Staněk [2] proved that the problem remains $\mathcal{NP}$-hard even if the guest graph $G = (V, E)$ is restricted to be a tree. Moreover, it is very unlikely that the problem is $\mathcal{NP}$-hard, because $\mathcal{NP} = \mathcal{P}$ would follow otherwise. **Is the problem solvable in polynomial time if both the guest and the host graph are binary regular trees (like in Example 2)? Is it possible that this problem is neither $\mathcal{NP}$-hard nor that it belongs to the complexity class $\mathcal{P}$? If yes: which complexity class does it belong to?**

References

[1] M.J. Luczak and S.D. Noble, Optimal arrangement of data in a tree directory, *Discrete Applied Mathematics* **121 (1–3)**, 307–315, 2002.

[2] E. Çela, R. Staněk, and J. Schauer, The data arrangement problem on binary trees, submitted for publication, available as `arXiv:1512.08404`.