Problems on tours and trees in combinatorial optimization

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This PhD thesis deals with three problems.

The data arrangement problem on regular trees (DAPT): The problem consists in assigning the vertices of a given graph G, called the guest graph, to the leaves of a d-regular tree T, called the host graph, such that the overall sum of the pairwise distances of pairs of leaves in T which correspond to edges of G is minimized. The problem was first considered by LUCZAK and NOBLE in 2002; they have shown that the DAPT is $\mathcal{NP}$-hard for every fixed d greater than or equal to 2.

We start by focusing on the special case of the DAPT where both the guest graph and the host graph are binary regular trees and provide a $\frac{203}{200}$-approximation algorithm for this special case. The solution produced by the algorithm and the corresponding value of the objective function are given in closed form. The analysis of the approximation algorithm involves an auxiliary problem which is interesting on its own, namely the *k-balanced partitioning problem* (*k-BPP*) for binary regular trees and particular choices of k. We derive a lower bound for the latter problem and obtain a lower bound for the original problem by solving h instances of the k-BPP, where h is the height of the host graph G. Subsequently, we provide a $\frac{585}{392}$-approximation algorithm for the special case of the DAPT, where both the guest graph and the host graph are d-regular trees for some fixed d greater than or equal to 2.

Finally, we show that the DAPT remains $\mathcal{NP}$-hard even if the guest graph is a tree. This issue was posed as an open question by LUCZAK and NOBLE.

The traveling salesman problem (TSP): Given a complete graph $G = (V, E)$ and non-negative distances d for every edge, the TSP asks for a shortest tour through all vertices with respect to the distances d. In our approach we try to exploit the impressive

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performance of current ILP solvers and work only with integer solutions. We stick to a very simple ILP model and relax the subtour elimination constraints only. The resulting problem is solved to integer optimality, violated constraints (which are trivial to find) are added and the process is repeated until a feasible solution is found. In order to speed up the algorithm, we pursue several attempts to find as many relevant subtours as possible. These attempts are based on the clustering of vertices with additional insights gained from empirical observations and random graph theory.

The minimum and maximum symmetric quadratic TSP: The *symmetric quadratic traveling salesman problem* (*SQTSP*) associates a cost with every triple of vertices traversed subsequently in the TSP tour. If the vertices correspond to points in the Euclidean plane and the costs are proportional to the turning angles of the tour, we speak of the *angular-metric traveling salesman problem* (*AngleTSP*).

We consider the SQTSP mainly from a computational point of view. In particular, we adopt the basic algorithmic idea used for the TSP and perform the separation of the classical subtour elimination constraints on integral solutions only. It turns out that this approach outperforms the standard fractional separation procedure known from the literature. We also test more advanced subtour elimination constraints introduced by Fischer and Helmberg in 2013 in both the integral and the fractional separation procedures.

Finally, we deal with the maximization variant *MaxSQTSP*. In contrast to the minimization counterpart it turns out that in this case some of the stronger subtour elimination constraints by Fischer and Helmberg now improve the performance of our approach. For the special case of *MaxAngleTSP* we can observe an interesting behavior: It can be shown that the sum of inner turning angles in an optimal solution always equals π if the number of vertices is odd. This implies that no subtour elimination constraints are needed to solve the problem. Moreover, we give a simple constructive polynomial time algorithm to solve the problem.

Keywords: Graph embedding; data arrangement problem; DAPT; approximation algorithm; partitioning; k-balanced partitioning problem; k-BPP; partitioning problem into sets of bounded cardinality; PPSBC; binary tree; d-regular tree; traveling salesman problem; TSP; subtour elimination constraint; symmetric quadratic traveling salesman problem; SQTSP; maximum quadratic symmetric traveling salesman problem; MaxSQTSP; angular-metric traveling salesman problem; AngleTSP; maximum angular-metric traveling salesman problem; MaxAngleTSP; quadratic integer program; linearization; MILP model; ILP model; ILP solver; random Euclidean graph